

MATH 281 - Foundations of Mathematics

Spring 2013

Instructor: Casey Douglas
Office: SH173
E-mail: cjdouglas@smcm.edu
Office Phone: x4369

Coffee Hours: T 2:00-3:00, Tr 3:00-4:00, F 10:00 - 11:00
and by appointment

TAs: Ashley Bonner (anbonner@smcm.edu)
Marissa Smith (msmith@smcm.edu)



“The product of mathematics is clarity and understanding. ... The real satisfaction from mathematics is in learning from others and sharing with others.” – ????

“We are servants rather than masters in mathematics.” – ????

“To speak freely of mathematics, I find it the highest exercise of the spirit; but at the same time I know that it is so useless that I make little distinction between a man who is only a mathematician and a common artisan. Also, I call it the most beautiful profession in the world; but it is only a profession.” – ????

“God exists since mathematics is consistent, and the devil exists since its consistency cannot be proved.” – ????

Course Content

This class serves as an introduction to the basic nuts and bolts of mathematics. How does mathematics work? What does it mean to say that a given statement is true, and how can you verify such a claim? While it may seem absurd to ask such basic questions *now*, after nearly 13 years of math classes, this sequencing mirrors historical events. Limits and derivatives were being used for nearly two centuries before good definitions were developed, for instance, and various cultures talked about infinity hundreds of years before Georg Cantor furnished a precise treatment of it.

One might also note that Lucas' *Star Wars* franchise began mid-story, with Episode IV (and one would do well to forget that the foundation-laying prequels were awful). In fact, it is precisely your familiarity with (and hopefully interest in) mathematics that makes you ready to learn something of its origin story. You've seen how math can be used and generalized, its now time to learn how it began. Thanks for taking the time to read this, by the way. Write your name along with the symbol \mathbb{Z} on a slip of paper or post-it note and hand it to me at the start of class on Friday.

Here is a (rough) list of topics we'll cover this semester:

- Proofs: What does it mean to prove something? Why do we prove things? What is an elegant proof?
- Set Theory: sets, power sets, combinations of sets
- Logical Statements: the language of mathematics
- Quantifiers: upside down A's and backward-facing E's (like these \forall , \exists)
- Induction: the power of logical dominoes
- Functions, Relations and Equivalence Classes: Clock arithmetic
- Problem solving skills: mo' money = mo' problems, but mo' math = mo' solutions

Textbook

Foundations of Higher Mathematics: Exploration and Proof (2nd Edition) by Fendel & Resek

Course Resources

Naturally, you have written resources to help you with the course material, some pre-existing (your textbook and class handouts) and some that you will produce as the course progresses (your blog entries and your homework). Arguably the most important resources at your disposal are people.

First and foremost you have each other. You are encouraged to work on assignments with others, and, in fact, you will often be organized into groups to collaborate on activities. Secondly, you have wonderful TAs Marissa Smith and Ashley Bonner. Lastly, you have me. Please make use of our office hours and review sessions.

Assignments and Blogging

There will be three different types of assignments: the problem(s) of the week, homework problems, and proofs. At the start of every week I will post a Problem of the Week in the math wing. Please stop by and read the problem. Solutions are due one week later. Written proofs will be assigned about once a week and collected in class. You will be graded on how complete and understandable your proofs are. Lastly, typical homework problems will also be assigned and collected on a weekly basis; these will be given out during class and/or via e-mail.

For your blog entries, consult the "Guide to Writing a FOM Blog" which can be found on my blog, <http://mathematicalypse.wordpress.com/category/fom-2013/>.

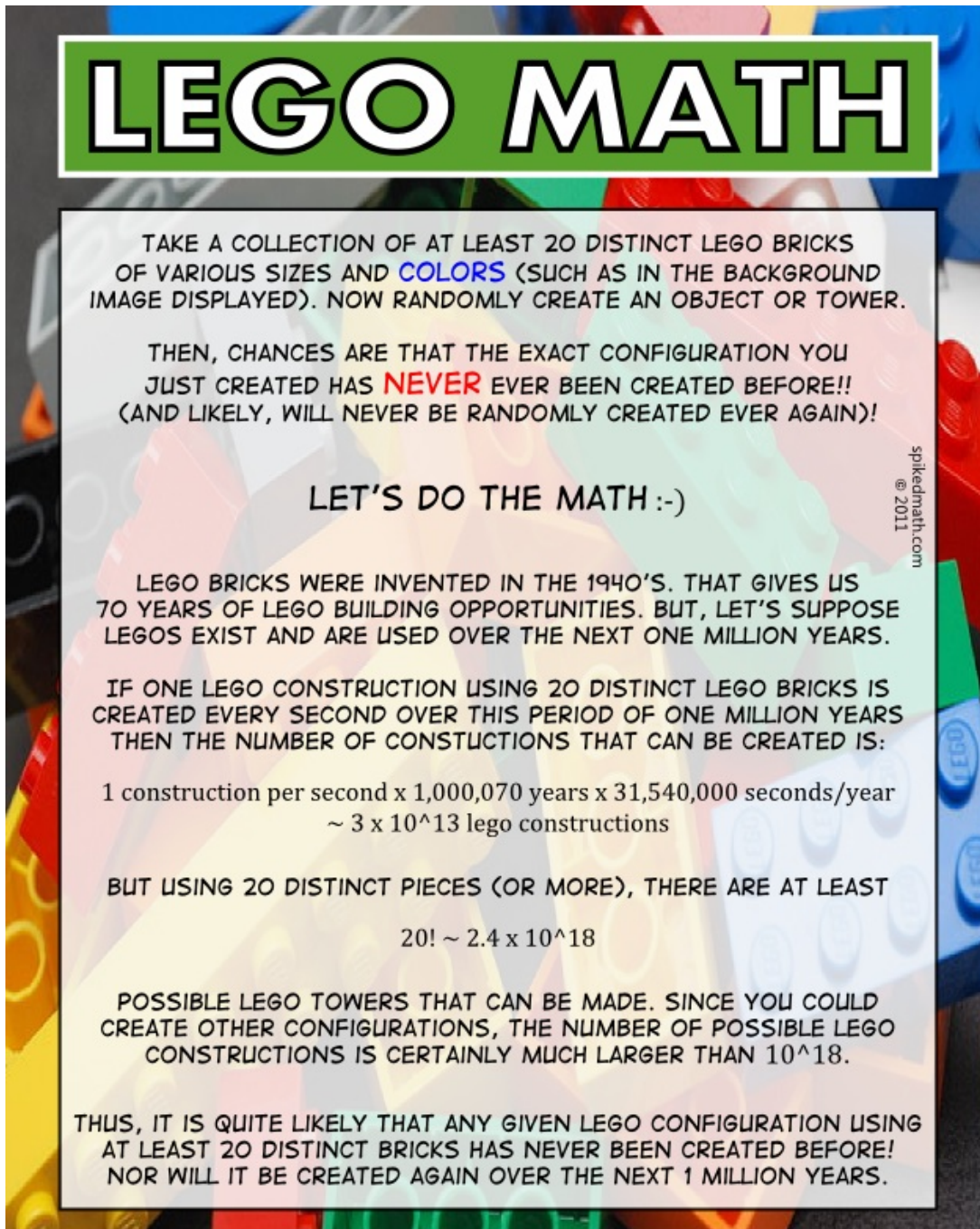
Grades and Tentative Exam Dates Grades will be computed as follows:

- FOM Blog: 20% (all semester)
- Assignments (proofs and problems): 30% (all semester)
- Class Participation: 15% (all semester)
- Midterm: 15%
- Take-home Final: 20%

I will use Blackboard to keep track of your grades.

Students with Disabilities

Any student with a disability requiring accommodations in this class is encouraged to contact me. Students with a disability may also wish to contact Lenny Howard in the Office of Academic Services.



LEGO MATH

TAKE A COLLECTION OF AT LEAST 20 DISTINCT LEGO BRICKS OF VARIOUS SIZES AND COLORS (SUCH AS IN THE BACKGROUND IMAGE DISPLAYED). NOW RANDOMLY CREATE AN OBJECT OR TOWER.

THEN, CHANCES ARE THAT THE EXACT CONFIGURATION YOU JUST CREATED HAS **NEVER** EVER BEEN CREATED BEFORE!! (AND LIKELY, WILL NEVER BE RANDOMLY CREATED EVER AGAIN)!

LET'S DO THE MATH :-)

LEGO BRICKS WERE INVENTED IN THE 1940'S. THAT GIVES US 70 YEARS OF LEGO BUILDING OPPORTUNITIES. BUT, LET'S SUPPOSE LEGOS EXIST AND ARE USED OVER THE NEXT ONE MILLION YEARS.

IF ONE LEGO CONSTRUCTION USING 20 DISTINCT LEGO BRICKS IS CREATED EVERY SECOND OVER THIS PERIOD OF ONE MILLION YEARS THEN THE NUMBER OF CONSTRUCTIONS THAT CAN BE CREATED IS:

1 construction per second x 1,000,070 years x 31,540,000 seconds/year
 $\sim 3 \times 10^{13}$ lego constructions

BUT USING 20 DISTINCT PIECES (OR MORE), THERE ARE AT LEAST

$20! \sim 2.4 \times 10^{18}$

POSSIBLE LEGO TOWERS THAT CAN BE MADE. SINCE YOU COULD CREATE OTHER CONFIGURATIONS, THE NUMBER OF POSSIBLE LEGO CONSTRUCTIONS IS CERTAINLY MUCH LARGER THAN 10^{18} .

THUS, IT IS QUITE LIKELY THAT ANY GIVEN LEGO CONFIGURATION USING AT LEAST 20 DISTINCT BRICKS HAS NEVER BEEN CREATED BEFORE! NOR WILL IT BE CREATED AGAIN OVER THE NEXT 1 MILLION YEARS.

spikedmath.com
© 2011